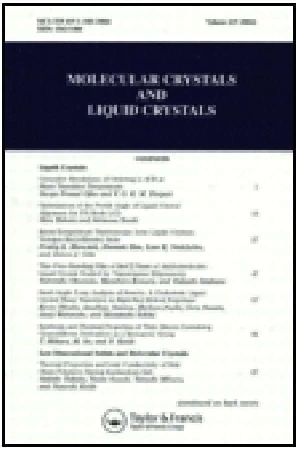

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Optical Transmission Model for Thin Two-dimensional Layers

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OPTICAL TRANSMISSION MODEL FOR THIN TWO-DIMENSIONAL LAYERS

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A simplified simulation method to calculate the transmission and diffraction of oblique incident light through a thin two-dimensional layer is presented. The algorithm, based on a ray-tracing technique, uses the Extended Jones-calculus and neglects diffraction within the layer. The algorithm is applied to an in-plane switching liquid crystal display. The transmission at the display surface and the diffracted field at a large distance from the display are compared with the more accurate reduced grating method. When the thickness of the layer is sufficiently small and the variation in liquid crystal orientation is slow, our method gives accurate results.

Keywords: diffraction; in-plane switching; liquid crystal display

INTRODUCTION

Calculating the optical transmission of a plane wave through a liquid crystal (LC) medium is a difficult task. For one-dimensional layers, simple matrix formalisms known as the Jones-calculus [1] and the Berreman-method [2] were developed years ago. These methods take phenomena such as

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refraction and birefringence into account and are sufficient for one-dimensional layered structures such as liquid crystal devices and anti-reflection coatings.

In two- and three-dimensional media, propagation is more complicated due to diffraction effects leading to refraction or scattering. Therefore, more precise calculation algorithms based on the finite-difference time-domain algorithm [3], beam-propagation [4] or the reduced-grating method (RGM) [5–7] have been developed. A drawback of these methods is that the calculations are time-consuming and require a lot of computer memory. Former studies made a comparison of these methods using the Jones-calculus for perpendicular incident light at an in-plane switching liquid crystal display (IPS-LCD). Transmission [8] at the top surface of the two-dimensional layer as well as the diffracted field [9] at a large distance of the display were studied.

In this article, the investigation is expanded to oblique light incidence using the Extended Jones-calculus [10,11] and comparing the results with the RGM. The algorithm for the propagation through the LC layer is explained in detail and the possibilities and limitations of the simplified method are examined.

SIMULATION MODEL

The simulation model starts from a two-dimensional LC medium that is invariable along the y-axis with the director orientation given on a regular rectangular grid in the x,z plane, as in the example on Figure 1. The optical parameters of the uniaxial LC are given by the ordinary and extra-ordinary refractive indices n_o and n_e. To assume the LC properties homogeneous inside each grid box, the grid must be sufficiently fine. The director

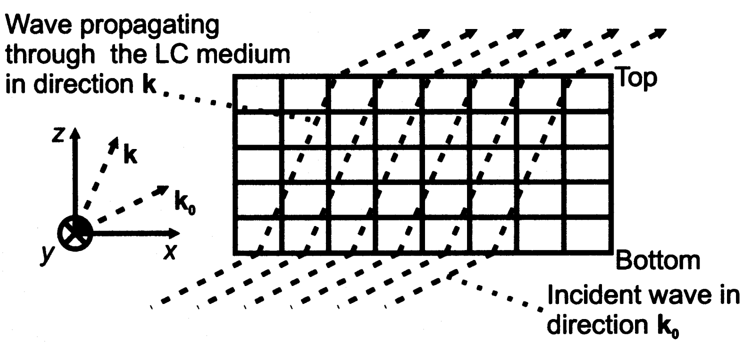

FIGURE 1 Basic principle of the simulation algorithm.

orientation of the LC molecules inside each rectangle is determined by the tilt angle θ (between the director and the x,y plane) and the twist angle φ (between the x-axis and the projection of the director on the x,y plane). Our goal is to calculate the propagation of an obliquely incident plane wave through the LC layer. The propagation direction of the incident plane wave in air, given by the wave vector $\mathbf{k_0}$, is determined by the inclination angle θ_k (angle with the z-axis) and the azimuth angle φ_k (angle between the x-axis and the projection of $\mathbf{k_0}$ on the x,y plane).

At the bottom surface of the LC layer refraction occurs. Because of refraction the propagation direction in air $\mathbf{k_0}$ is changed to a different direction $\mathbf{k}$ inside the medium, the inclination angle θ_k is changed according to Snell's refraction law. As refractive index for the LC medium, we choose the ordinary refractive index n_o, in accordance with the Extended Jones-calculus. In the case of small birefringence, for example $n_e - n_o = 0.097$, the difference in wave vector for the ordinary and extra-ordinary wave is negligible. The Extended Jones-calculus has been developed for one-dimensional media. To be able to use the Extended Jones-calculus we assume that in our structure the thickness of the layer measured along the z-axis is small compared to the lateral liquid crystal dimensions and not much larger than the wavelength λ of the light. When the variation of the LC director inside the medium is relatively slow, the Extended Jones-calculus provides a good approximation of the correct result [12]. In this case, we may calculate the propagation through the LC only taking into account birefringence and neglecting diffraction inside the layer. The algorithm is illustrated in Figure 1. In Figure 1 the plane of incidence containing the wave vectors $\mathbf{k}$ and $\mathbf{k_0}$ coincides with the x,z plane and $\varphi_k = 0$, but in general φ_k can vary freely. To handle a plane wave incident on the two-dimensional medium, a technique similar to ray tracing is used. The plane wave is constructed as a large number of rays parallel to the vector $\mathbf{k_0}$ in air or $\mathbf{k}$ inside the medium. From each of the grid boxes at the bottom of the layer a ray propagates towards the other side in the direction of $\mathbf{k}$. For each rectangle in which a ray passes on its way, a propagation matrix based on the Extended Jones-calculus is calculated. In order to calculate the Jones-matrix for one grid box, the length of the path that the ray makes through this box has to be taken into account. By doing so, the variation of the LC in the x-direction is neglected. If the variation of the liquid crystal director along the x-axis is sufficiently slow, this is a good approximation. The propagation matrices are calculated according to the formula for the Extended Jones-calculus presented by A. Lien [13].

The result of the calculation is a series of Jones-vectors providing the electric field at the top surface as a function of the lateral position x and from this the propagation of the field through the analyzer can be calculated. The intensity of the electric field provides the near field transmission. Combining the complex Jones-representation of the electric field and diffraction theory gives the far field.

TRANSMISSION NEAR THE LAYER SURFACE

The simulation method has been tested by applying it to an IPS-LCD. An IPS-LCD consists of a thin liquid crystal (LC) layer between two glass substrates. The transparent electrodes form an interdigitated pattern of positive and negative stripes on the bottom glass substrate parallel to the y-axis. The LC layer has a thickness of 4 μm, the width of the electrodes is 6 μm and the gap in between is 18 μm. The rubbing direction of the alignment layers, which fixes the orientation of the molecules at the glass substrates, makes an angle of 10° with the y-axis. The pretilt, the angle of the director with the glass surface is about 1°. The liquid crystal material is Merck ZLI-4792 ($k_{11} = 13.2$ pN, $k_{22} = 6.5$ pN, $k_{33} = 18.3$ pN, $\varepsilon_{par} = 8.3$, $\varepsilon_{ppd} = 3.1$, $n_o = 1.4793$ and $n_e = 1.5763$). When a voltage is applied between the electrodes, the calamitic LC molecules tend to orient themselves along the field lines of the two-dimensional electric field that appears. The director simulations for the LC medium were performed using the software package 2dimMOS [14]. The polarizer at the bottom of the LCD is oriented perpendicular to the rubbing direction and the analyzer at the top parallel. The optical simulations presented in this article, use a monochromatic incident plane wave with a wavelength of 632.8 nm and the incident direction is given by an inclination and azimuth of 25°.

We verify our simulation results by comparing them with the more accurate RGM [5–7]. First, the transmission of an unpolarized plane wave through the IPS-LCD described above, with polarizer and analyzer present, is examined. The Extended Jones-calculus can only handle polarized light. To simulate an unpolarized incident plane wave we decompose the incoming plane wave in two plane waves with an intensity equal to half of the intensity of the unpolarized plane wave and mutually orthogonal polarizations. The intensity of the transmission of the unpolarized plane wave can be calculated as the sum of the transmission of the two polarized plane waves. We investigate the field near the top surface along the x-axis after passing the analyzer. Figure 2 shows the intensity of the transmission at the surface relative to the intensity of the incident unpolarized plane wave when voltages of 5, 10, 15, 20 and 25 V are applied between the positive and the negative electrodes. The plotted region starts in the middle of one electrode and ends in the middle of the next one. The graph shows that the overall correspondence between the method using the Extended Jones-calculus (EJC) and the RGM is quite good. At 5 V, the applied voltage is just above the threshold voltage and only at the edges of the electrodes

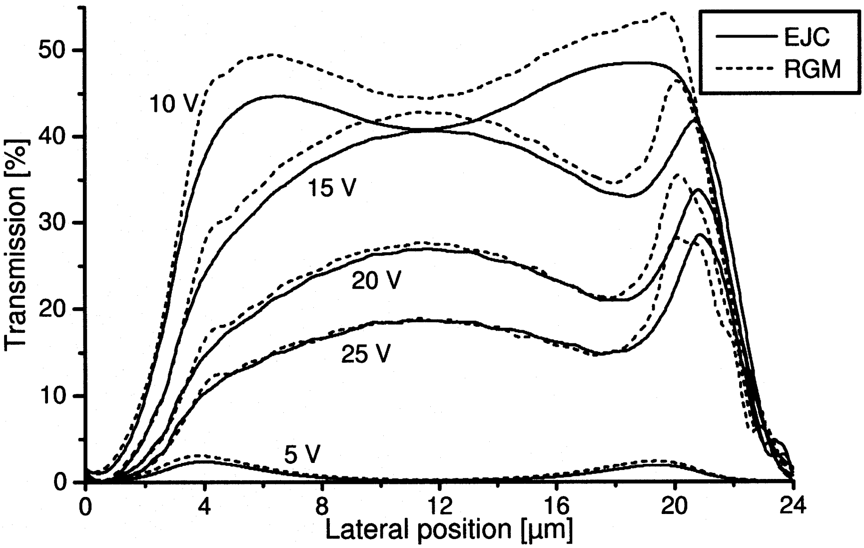

FIGURE 2 Transmission through one period of an IPS-LCD in function of the lateral displacement for different applied voltages. The plotted region starts in the middle of one electrode and ends in the middle of the next one.

the electric field is strong enough to reorient the liquid crystal. The transmission reaches a maximum at 10 V and decreases for higher voltages. In the bulk between the two electrodes, the electric field is parallel to the glass surface. In this area the director is almost uniform and the correspondence between both methods is excellent. At higher voltages the refractive index variations above the electrodes increase and diffraction within the LC becomes important. This explains the deviation at the right-hand side of the graph for higher voltages. The asymmetry in the curves is due to the oblique incidence of the light. Because of the 1° pretilt of the molecules, a small asymmetry remains present for perpendicular light incidence.

FAR FIELD RESULTS

In display applications, the variation of the transmitted electric field at the top surface is interesting, however more important is the transmission seen by the human eye at a distance from the display. This far field can be calculated as the diffraction pattern of the electric field at the surface. The LC layer in the IPS-LCD as described above is periodical, with a period $\Lambda = 24\,\mu\text{m}$. Due to the periodicity of the LC layer, distinct diffraction orders appear and the Fraunhofer approximation of diffraction can be implemented by a complex fast fourier transform of the electric field components at the surface. Figure 3 shows the intensity of the diffraction orders for the Jones-method and the RGM for applied voltages of 5, 15 and 25 V. The graph shows a very good correspondence for the central diffraction order (order 0). The central diffraction order propagates along the direction $\mathbf{k_0}$ and corresponds with the average value of the complex field at the top surface of the IPS-LCD. According to the Bragg-condition for diffraction, the higher orders propagate in the directions $\mathbf{k_n} = \mathbf{k_0} + n.\mathbf{K}$ ($n = \pm 1, \pm 2, \ldots$), with $\mathbf{K}$ a vector along the x-direction and a length of $2\pi/\Lambda$. The intensities for most diffraction orders correspond quite well, but for a limited number of points a large deviation between the different methods is visible. The cause for this deviation is the phase variation of the electric field at the top surface. From Figure 2 we can conclude that the method using the Jones-calculus is accurate in predicting the amplitude of the electric field. However the calculation of the phase variation of the electric field before propagating through the analyzer, shown in Figure 4, is less good. To create this graph, the linear increasing phase due to oblique incidence was substracted. By doing this, the phase function becomes periodical and the graph is better understandable. Even for quite low

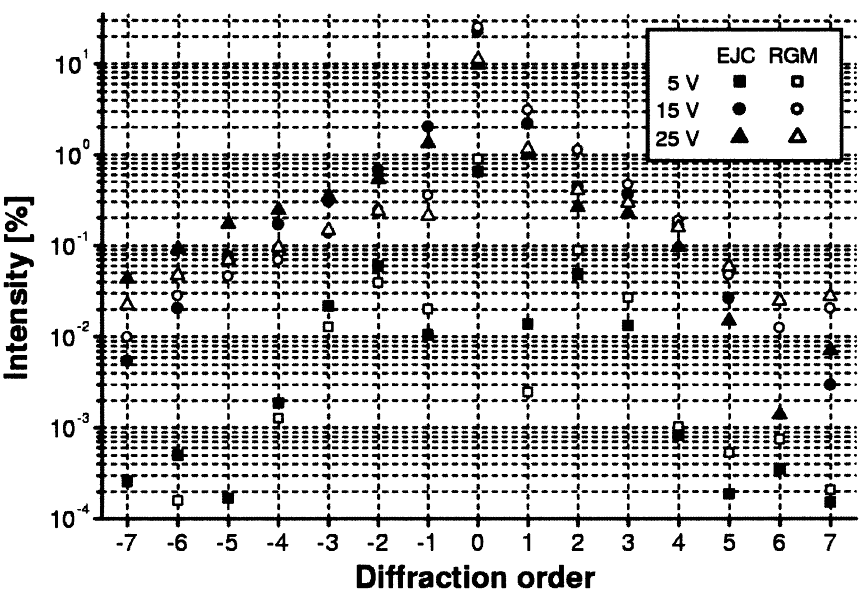

FIGURE 3 Intensity of the diffraction orders in the Fraunhofer diffraction pattern of an IPS-LCD.

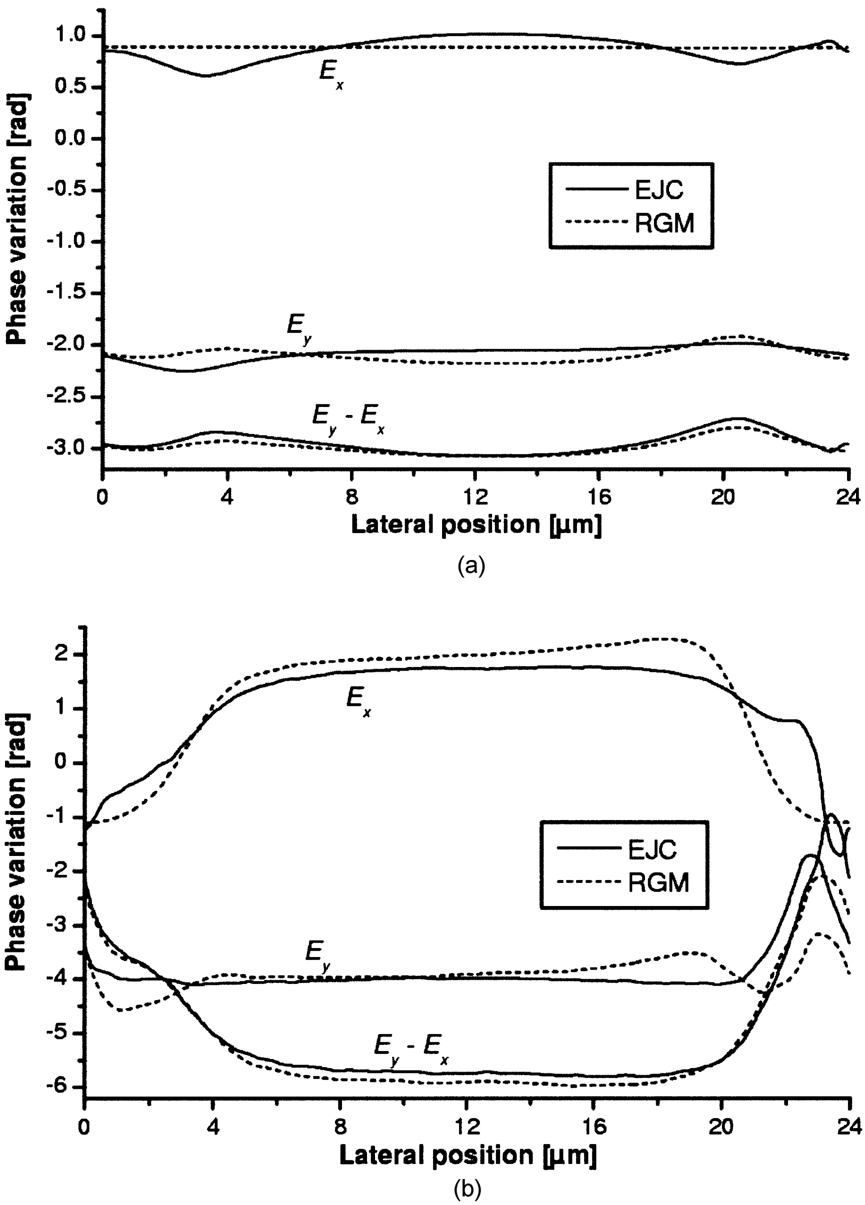

FIGURE 4 Variation of the phase of the electric field components E_x, E_y and the relative phase $E_y - E_x$ before propagation through the analyzer at the top surface of the IPS-LCD for an applied voltage of (a) 5 V and (b) 25 V.

voltages as in Figure 4a, there is a small deviation in the phase variation and for increasing voltage this deviation becomes larger. For the analysis of this imprecision, the phase of the electric field is divided in two parts, an absolute and a relative phase variation.

The origin of the relative phase lies in the birefringence of the liquid crystal. When propagating through a birefringent medium, a plane wave is split up in two modes which propagate with a different speed. This retardation is responsible for the change in polarization state of the plane wave and is directly related to the transmission of a liquid crystal cell between crossed polarizers. In Figure 4, the relative phase corresponds with the difference between the phases of E_y and E_x and is indicated as $E_y - E_x$. Although the phases of E_x and E_y differ for both methods at high voltages, the relative phase $E_y - E_x$ corresponds very well. This corresponds with the fact that birefringence is correctly taken into account by the Jones-calculus and explains the good match between both methods in Figure 2.

The absolute phase variation of the electric field is related to the propagation distance of the rays in the medium. If diffraction is absent in the medium, the absolute phase variation is the same for all rays that propagate through the medium which corresponds with the assumption we have made when explaining the algorithm. Diffraction is a combination of scattering and focusing of a plane wave. The differences in the absolute phase variation are mainly due to focusing of the wave. On the right-hand side of Figure 4 large deviations of the phase of E_x and E_y are visible between both methods due to the rapid director changes in the liquid crystal. An example where birefringence is absent and focusing is important is the propagation through a thin lens as illustrated in Figure 5. After propagation through the lens, a perpendicular incident plane wave has a spherical wave front. At increasing distances the intensity increases above the center of the lens and the phase front becomes steeper. Finally, the wave focuses into one point. The RGM handles this correct, but in the method using the Extended Jones-calculus the rays will not focus but keep propagating in the same direction with a fixed phase front.

LIMITATIONS OF THE METHOD

To ensure that the method with the Extended Jones-calculus gives accurate results diffraction effects have to be negligible. Therefore, the layer should be thin enough so that the focusing has not yet occurred and the lateral liquid crystal variations should be low within a distance in the order of one wavelength. In the example of the thin lens, the field is calculated correctly immediately after the lens, but at larger distances focusing is no longer negligible. Although the right-hand side of the liquid crystal

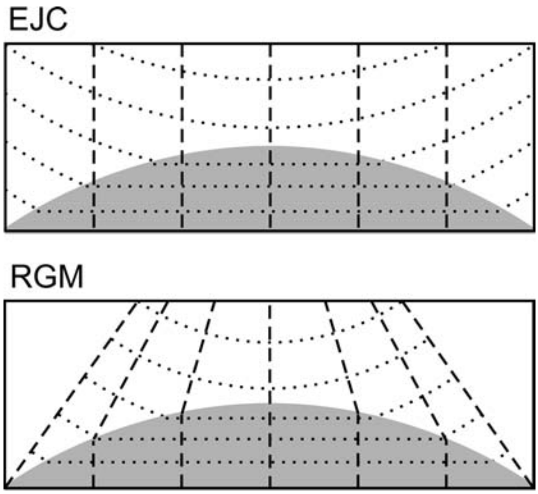

FIGURE 5 Simple example of perpendicular incidence on a thin lens where the method using the Jones-calculus fails due neglection of focusing effects.

inside the IPS-LCD is practically symmetric to the left-hand side, the transmission curves in Figure 2 show an asymmetry. Also the differences in the absolute phase are mainly present on the right-hand side. Therefore, a quantitative way to evaluate the lateral variations is explained. In fact the variations of the refractive index of the medium are important. For liquid crystals this is rather complex since the material is birefringent. When a plane wave propagates through a birefringent medium a superposition has to be made of the ordinary and the extra-ordinary wave. The first wave feels the ordinary refractive index n_o of the liquid crystal which is constant. The extra-ordinary wave feels an effective refractive index n_{eff}. n_{eff} depends on the angle between the direction of propagation $\mathbf{k}$ and the local orientation of the liquid crystal director and has a value between n_o and n_e. Figure 6 represents the effective refractive index of the extra-ordinary wave corresponding with the propagation direction $\mathbf{k}$ for the molecules in the middle section of the liquid crystal for the same voltages as in Figure 2. For perpendicular incidence the curves are symmetric, but due the oblique incidence the variations of the refractive index are much larger at the right-hand side and we can expect the largest differences in that region for the transmission and the phase of the electric field.

A similar ray-tracing technique has been demonstrated in [15] to calculate iso-transmission and iso-contrast plots of an electrically-induced optical compensation display mode, but only the average transmission

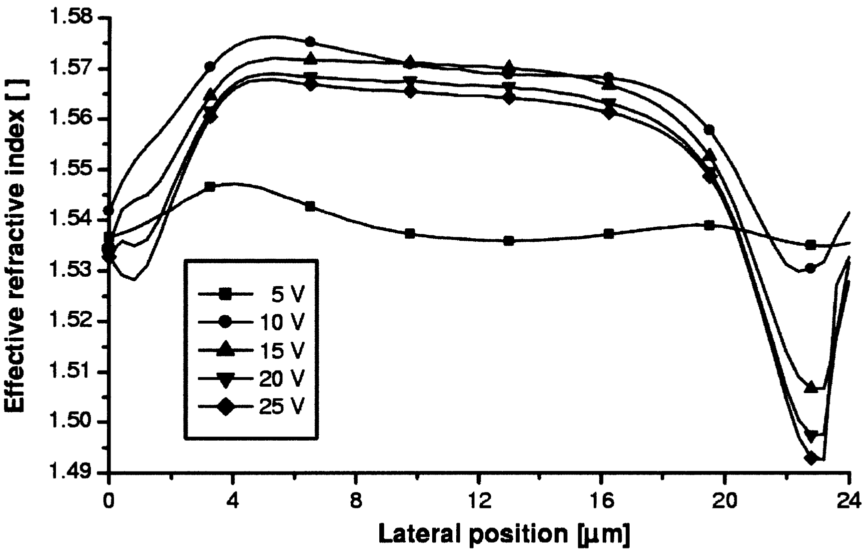

FIGURE 6 Variation of the effective refractive index of the extra-ordinary wave for the molecules in the middle of the liquid crystal layer and propagation direction k.

and contrast were used to calculate the far field results. The limitations of the method as in the above section were not discussed.

CONCLUSIONS

A simple and fast simulation algorithm to calculate the transmission through thin two-dimensional layers is presented. The method is based on neglecting the diffraction inside the thin layer and shows good results in predicting the transmission at the top surface of an IPS-LCD. Due to focusing of the light, the calculation of the phase of the electric field is less accurate in the regions with the largest changes in refractive index. The far field results are good but sensitive to the phase deviations. The calculation method using the Extended Jones-calculus is useful in most display applications which work with thin layers and where the applied voltages are usually low. In the example of IPS-LCDs, the device is generally operated at voltages below the value where the average transmission starts to decrease, which is in our configuration around 10 V. Moreover, the intensity of the higher diffraction orders are at least 10 times lower than the intensity of the central order, so their influence is negligible. Finally, the algorithm is fast and easily expandable to the three-dimensional case.

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